

## Fermat's "Challenge Problems" of 1657

In 1657, Fermat posed various "challenge problems" by letter, writing to other famous mathematicians of the time in Latin and/or French. See *Oeuvres de Fermat*, volume 2, starting with page 332. Here are a few of them:

First problem, dated January 3, 1657: to find (positive perfect) cubes with the property that the sum of the number and of all of its proper divisors is a perfect square. Fermat proposed this one to "Wallis and the other English mathematicians" "with hearty commendations of the messenger, Thomas White." He said that if the English, Dutch, and "Gallic Belgians" couldn't solve it, he [Fermat] would. Fermat also gave an example:

$$7^3 = 343, 343 + 49 + 7 + 1 = 400 = 20^2$$

and asked if there are any other solutions.

Second problem (superficially quite similar, also in the same letter): to find a perfect square such that the sum of the number and all of its proper divisors is a perfect cube.

Next challenge, dated February, 1657: To find positive integer solutions to the equation

$Nx^2 + 1 = y^2$ , for given a positive nonsquare  $N$ . Fermat gives the example of  $N = 3$ , for which he gives the solutions  $x = 1, y = 2$ , since  $3 \times 1^2 + 1 = 2^2$ , and  $x = 4, y = 7$ , since  $3 \times 4^2 + 1 = 7^2$ . He then asks for the smallest solutions with  $N = 61$  or  $N = 109$ .